Shafts

A. SHAFTS

1.0 INTRODUCTION

Shafts are used to transmit motion, torque and/or power in any combination.They are also subject to lateral loads. These can be constant or fluctuating. The sizing of shafts, therefore, is usually determined as a function of torsionally Induced stresses (shear stresses), bending stresses (tensile or compressive stresses) and the nature of the load (constant or fluctuating).

2.0 DETERMINATION OF STRESSES FOR SOLID CYLINDRICAL SHAFTS

(a) Nomenclature

Let d = shaft diameter, in.
$H.P.$ = horsepower
K_m = shock factor for bending loads
K_t = shock factor for torsional loads
M = bending moment, In-lbs.
N = revolutions per minute (RPM)
S = shear stress, lbs/in²
S_{max} = maximum allowable shear stress, lbs/in²
T = torque, in-lbs.
T_{max} = maximum allowable torque, In-lbs.

(b) Relation Between Torque and Horsepower

$$T = \frac{63{,}025\ (H.P.)}{N} \qquad (1)$$

(c) Torsional Loading

For shafts under steady torsional loads (and no bending loads),

$$d = \left(\frac{5.09\ T}{S}\right)^{1/3} \qquad (2)$$

In the particular case of a shaft in which S_{max} = 12,000 psi (for example in the case of #303 stainless steel and a gradually applied load), equations (1) and (2) can be combined to yield:

$$T_{max} = 2353\ d^3 \qquad (3)$$

and

$$(H.P.)_{max} = 0.037\ d^3 N \qquad (4)$$

Table 1 shows maximum safe torsional loads based on #303 stainless steel with S_{max} = 12,000 psi and a gradually applied load:

TABLE 1

MAXIMUM SAFE TORSIONAL LOADS BASED ON #303 STAINLESS STEEL WITH S_{max} = 12,000 PSI AND GRADUALLY APPLIED LOAD

Shaft Diameter (inches)	1/16	3/32	1/8	5/32	3/16	7/32	1/4	5/16	3/8	1/2
T_{max}	0.56	1.89	4.60	8.93	15.64	24.38	36.77	71.46	124.1	295

The maximum torques in this table can be converted to horsepower for a given value of N (RPM) by the power Nomogram given in the Designer Data Section of this book.

(d) Combined Torsional and Bending Loads

Combined bending and torsion arises as a result of component weight, belt tension, gear-tooth forces, etc. In that case shaft size can be determined from the equation:

$$d = \left[\frac{5.1}{S_{max}} \left((K_m M)^2 + (K_t T)^2\right)^{1/2} \right]^{1/3} \qquad (5)$$

where the values of the shock factors (K_m, K_t) are given in the following table:

TABLE 2
SHOCK FACTORS VS. TYPE OF LOAD

Type of Load	K_m	K_t
Gradually applied load	1.0	1.0
Minor shocks, sudden load	1.4—2.0	1.0—1.5
Heavy shocks, sudden heavy loads	2.0—3.0	1.5—3.0

If we arbitrarily consider only minor shock loads and assume that $K_m = K_t = (2)^{1/2} \cong 1.41$ and (as before) S_{max} 12,000 psi, the combined allowable bending and torsional moments can be related to shaft size as shown in the following table:

TABLE 3
TABLE OF COMBINED LOADING OF SHAFTS
(Based on #303 stainless Steel With S_{max} = 12,000 psi. end Minor Shocks)

Shaft Dia.	Moment	Inch-Pounds										
1/16	Tortion	0	0.040	0.079	0.119	0.159	0.198	0.238	0.277	0.317	0.356	0.396
	Bending	0.396	0.394	0.388	0.378	0.362	0.342	0.317	0.283	0.236	0.173	0
3/32	Tortion	0	0.134	0.268	0.402	0.536	0.670	0.804	0.938	1.07	1.21	1.34
	Bending	1.34	1.33	1.31	1.28	1.23	1.16	1.07	0.953	0.803	0.572	0
1/8	Tortion	0	0.325	0.650	0.975	1.30	1.63	1.95	2.28	2.60	2.92	3.25
	Bending	3.25	3.24	3.20	3.09	2.98	2.86	2.60	2.31	1.95	1.43	0
5/32	Tortion	0	0.632	1.26	1.89	2.52	3.16	3.79	4.42	5.05	5.68	6.32
	Bending	6.32	6.28	6.18	6.02	5.79	5.47	5.06	4.51	3.78	2.76	0
3/16	Tortion	0	1.11	2.22	3.33	4.44	5.55	6.66	7.77	8.88	9.99	11.1
	Bending	11.1	10.9	10.8	10.5	10.3	9.56	8.82	7.86	6.58	4.73	0
7/32	Tortion	0	1.72	3.44	5.16	6.88	8.60	10.3	12.0	13.8	15.5	17.2
	Bending	17.2	17.1	16.8	16.5	15.8	14.8	13.8	12.7	10.7	7.53	0
1/4	Tortion	0	2.60	5.20	7.80	10.4	13.0	15.6	18.2	20.8	23.4	26.0
	Bending	26.0	25.8	25.4	24.8	23.8	22.5	20.8	18.5	15.6	11.3	0
5/16	Tortion	0	5.06	10.1	15.2	20.2	25.2	30.3	35.4	40.4	45.5	50.5
	Bending	50.5	50.3	49.6	48.2	46.3	43.7	40.4	36.1	30.4	2.19	0
3/8	Tortion	0	8.77	17.5	26.3	35.1	43.8	52.6	61.4	70.2	78.9	87.7
	Bending	87.7	87.4	85.9	83.7	80.4	75.9	70.2	62.6	52.6	38.4	0
1/2	Tortion	0	20.6	41.6	62.4	83.2	104	124.8	145.6	166.4	187.2	208.0
	Bending	208	207.4	205	200	192	181	168	151	127.5	93.5	0
5/8	Tortion	0	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4	406.2
	Bending	406.2	404.2	398	387.5	372.3	351.9	325.1	290.2	244.0	177.5	0
3/4	Tortion	0	70.2	140.4	210.6	280.8	351.0	421.2	491.4	561.6	631.8	702
	Bending	702	698.4	687.8	669.6	643.3	607.9	561.5	501.3	421.1	305.9	0

If AISI 1213 steel is used, S_{max} = 8000 psi. and the diameters given in the tables have to be increased by 15%.

Example

A 1/50th H.P. motor drives a shaft rotating at 315 RPM and is subject to a bending moment of 6 in-lbs, the load application being sudden. Determine shaft size for a #303 stainless steel shaft.

From equation (1),

$$T = \frac{63{,}025\ (H.P.)}{N} = \frac{(63{,}025)\ (0.02)}{(315)} = 4 \text{ in-lbs.}$$

Referring to Table 3 (Combined Loading), we find that the combination of an allowable torsional moment in excess of 4 in-lbs. (actual 4.44) together with an allowable bending moment in excess of 6 in-lbs. (actual 10.3) first occurs for a 3/16" shaft diameter.

If there were no bending moment and the load were to be applied gradually ($K_m = K_t = 1$) Table 1 shows that a 1/8” shaft diameter would be sufficient.

Precision ground stainless steel shafts in undersize, nominal and oversize dimensions, as well as cold-drawn precision low-carbon steel shafts are available from SDP, as featured in our catalog. A new line of case hardened steel (C1060), case hardened stainless steel (440C) and thru hardened stainless steel (416) is being offered for use with bronze and needle bearings.

(e) Combined Torsional and Bending Loads for Fluctuating Loads With Stress Reversal

In such cases fatigue failure governs the design. Various failure theories have been proposed (e.g. Soderberg, Mises—Hencky etc.). All of these result in formulas involving both the average loading and the reversing component of the loading. Agreement with experiment varies depending on the nature of the material and failure mode. The equations resulting from these several theories also involve yield-point stresses and endurance limits and the various considerations governing the applicability of a particular set of equations extends beyond the scope of this discussion. For a fuller treatment the reader is referred to the following literature:

(i) R.M. Phelan “Fundamentals of Mechanical Design”, Third Edition, McGraw- Hill, New York,
N.Y., 1970. Chapter 6.
(ii) J.E. Shigley: “Mechanical Engineering Design”, Third Edition, McGraw-I-fill, New York, N.Y.,
1977, Chapter 13.
(iii) M.F. Spotts: “Design of Machine Elements”, Third Edition, Prentice-Hall, Englewood Cliffs,
New Jersey, 1961, Chapter 3.

3.0 HOLLOW CYLINDRICAL SHAFTS

In the case of hollow cylindrical shafts under torsion and/or bending, we can proceed as follows. Suppose a hollow shaft has an outside diameter, d_0, and an inside diameter, d_j, To size such a shaft for torsional and/or bending loads, we can convert the design calculation to that for an equivalent solid shaft of the same material by noting that the stress is inversely proportional to the ratio of moment of inertia (of the shaft cross-section) to shaft radius.
Equating this ratio for both the hollow shaft and the equivalent solid shaft of diameter d_{eq} we have:

$$\frac{\pi(d_0^4 - d_1^4)}{32 d_0} = \frac{\pi d_{eq}^3}{32}$$

Solving for d_{eq}, we have:

$$d_{eq} = \left(\frac{d_0^4 - d_1^4}{d_0}\right)^{1/3} \qquad (6)$$