3.0 BEARING SELECTION

Bearing selection represents a compromise among many factors including the nature of the applica tion, performance requirements and cost. A useful bearing-selection chart, which summarizes the principal considerations involved, has been given by A.O. DeHart and is reproduced in Table 1. For more details, which are beyond the scope of this presentation, the reader is referred to the technical literature.

4.0 BEARING LOADS

The first step in sizing a suitable ball bearing for a given application is the determination of the loads which it has to support. In this section we list some of the most frequently occurring mechanical configurations and the bearing loads imposed by them

(a) Radial Shaft Load Between Bearings

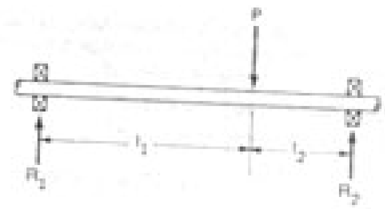


Figure 3 Radial Load Between Bearings

P = radial load

R1, R2 = bearing load

$l_1 . l_2$ = distance from Radial load to bearings

$$R1 = \frac{l_2 P}{l_1 + l_2} \quad (1)$$

$$R2 = \frac{l_1 P}{l_1 + l_2} \quad (2)$$

(b) Overhung Radial Load

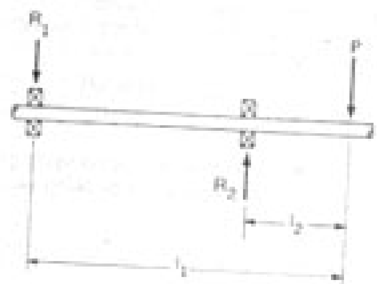


Figure 4 Overhung Radial Load

Notation same as in paragraph (a).

$$R1 = \frac{l_2 P}{l_1 + l_2} \quad (3)$$

$$R2 = \frac{l_1 P}{l_1 - l_2} \quad (4)$$

The maximum bearing load on either pulley shaft occurs when the belt is transmitting the maximum horsepower

(c) Flat belt Drives

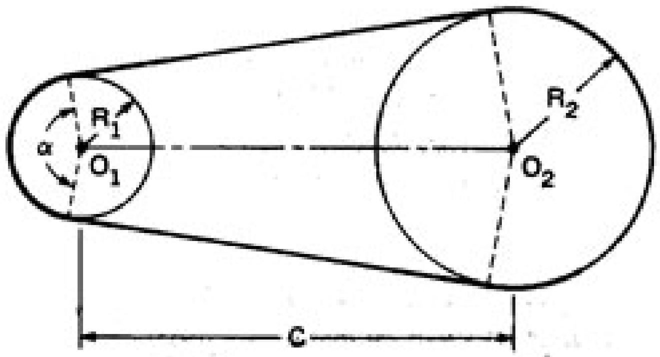


Figure 5 Belt Drive

α = angle of wrap of smaller pulley, radians (1 rad. = 57.3 deg.)
μ = coefficient of friction between pulley and belt
R_1 = radius of smaller pulley,in.
R_2 = radius of larger pulley, in.
c = center distance (O_1O_2), in.
$\phi = \sin^{-1} \left(\frac{R_2 - R_1}{C}\right)$; $0° <= \phi <= 90°$
H.P. = horsepower transmitted by belt
N_D = RPM of driving pulley
R_D = radius of driving pulley (i.e. either R_1 or R_2, dependinding on which is the driver)

The maximum bearing load on either pulley shaft occurs when the belt is transmitting the maximum horsepower (i.e. the belt would slip if the horsepower were Increased above this level). Under this condition the maximum bearing load is given by:

$$\text{Max. bearing load} = \frac{(63{,}025)\text{H.P.}}{N_D R_D} \left(\frac{e^{\mu a}+1}{e^{\mu a}-1}\right) \cos\varnothing \text{ lbs.} \qquad (5)$$

Note: In the case of chain drives the bearing load is often approximated by the pull on the tight side of the chain, the slack side being assumed tensionless.

(d) Unbalanced Rotors

(i) Tilted rotor or swashplate

α = tilt angle of rotor or swashplate
γ = density of rotor, lbs/in^3
ℓ = distance between bearings
r = radius of rotor, in.
t = thickness of rotor, in.
N = RPM of rotor.

Rotor is assumed to be uniform and with its center on the axis of rotation of the shaft.

R = bearing reaction at either bearing, lbs.

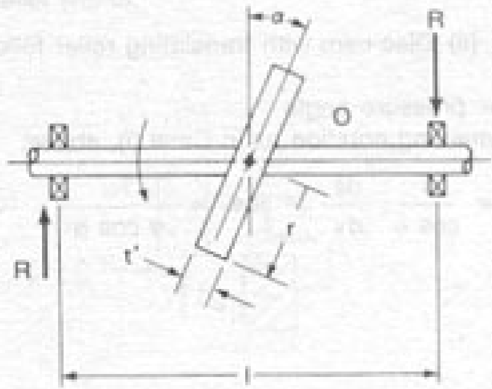


Figure 6 Tilted Rotor

$$R = \frac{N^2 r^4 t\gamma \sin 2\alpha}{89{,}633\,\ell} \text{ lbs} \quad (6)$$

(ii) Eccentric rotor

r, t, γ, N as in Case (i)
ℓ_1, ℓ_2 = distance from radial force, P, to bearings, in.
P = Radial force due to eccentricity, lbs.
e = eccentricity, in.
$R_{1,2}$ = bearing reactions, lbs.

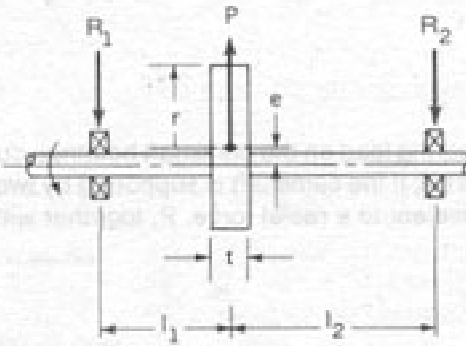


Figure 7 Eccentric Rotor

$$P = (8.925)(10)^{-5}\gamma r^2 N^2 te \text{ lbs.} \quad (7a)$$

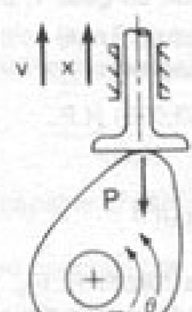
$$R_1 = \frac{P\ell_2}{\ell_1 + \ell_2} \text{ lbs.} \quad (7b)$$

$$R_2 = \frac{P\ell_1}{\ell_1 + \ell_2} \text{ lbs.} \quad (7c)$$

(e) Cams

(i) Disc cam and flat-faced translating follower

θ = cam rotation, radians (1 rad. = 57.3 degrees)
x = follower displacement (in.)
v = follower velocity (in/sec)
P = radial force on cam shaft, lbs.
ω = angular velocity of cam (rad/sec).



$$P = T\,\frac{d\theta}{dx} \text{ lbs.} = T\,\frac{\omega}{V} \quad (8)$$

where T = torque on cam transmitted by camshaft, in-lbs.

Figure 8 Disc Cam and Flat-faced

The bearing load on the camshaft bearings due to the load. P. can be determined according to Cases (a) or (b). if the camshaft is supported by two bearings.

(ii) Disc cam with translating roller follower

α = pressure angle
Remaining notation as in Case (i), above.

$$P = \frac{T}{\cos\alpha}\ \frac{d\theta}{dx}\ \text{lbs.} = \frac{T\omega€€€}{v\cos\alpha} \qquad (9)$$

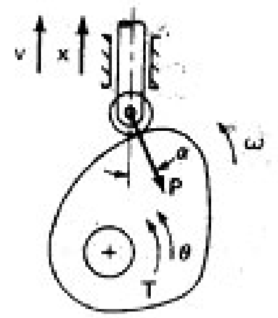


Figure 9 Disc Cam and Translating Roller Follower

The bearing load on the camshaft bearings due to the load, P, can be determined according to Cases (a) and (b), if the camshaft is supported by two bearings. Note that the force P in the above two cases is equivalent to a radial force, P. together with a torque about the cam axis.

(f) Spur Gears (External)

F = tangential force exerted by gear 1 on gear 2, lbs.
F_R = radial force exerted by gear 1 on gear 2, lbs.
R_1 = pitch radius of gear 1, in.
N_1 = RPM of gear 1
T_1 = torque on gear 1, in-lbs.
θ = pressure angle
H.P. = horsepower

$$F = \frac{(63{,}025)\ \text{H.P.}}{N_1 R_1} \qquad (10)$$

$$F_R = F \tan\theta \qquad (11)$$

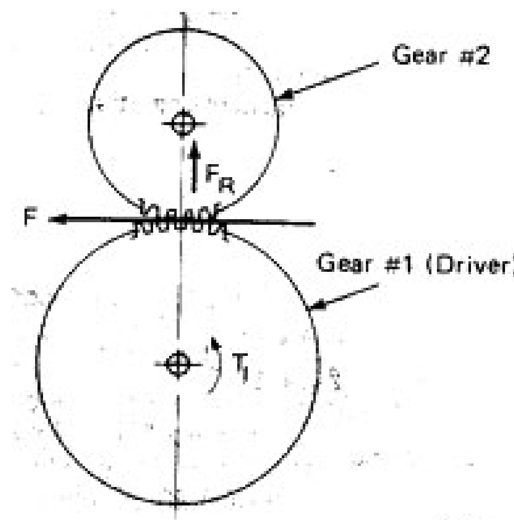


Figure 10 Spur Gears

Note that the forces F, F_R, which are exerted on gear #2, result in reaction forces of the same magnitude (but opposite direction) which act on gear #1. Hence magnitude of total radial load on gears

1 and 2 = $(F^2 + F_R^2)^{1/2}$ (12)

In the case of shafts with two bearings, the corresponding bearing loads are given in Cases (a) and (b). Equations (10, 11, 12) remain unchanged for Internal gears.

(g) Helical Gears

We consider here Only the case of helical gears on parallel shafts.

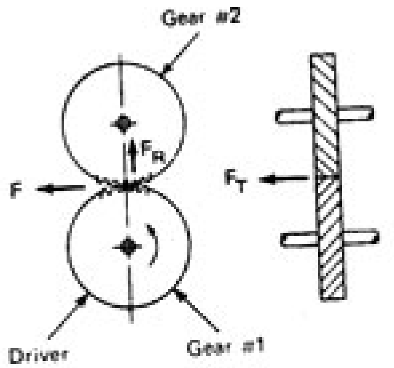


Figure 11 Helical Gears

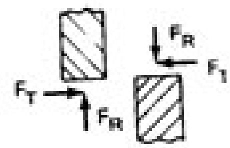


Figure 12 Directions of Radial and Thrust Forces in Plane Perpendicular to Shaft Axes

F = tangential force, lbs.
F_R = radial force, lbs.
F_T = thrust load, lbs.
H.P. = horsepower
N_1 = RPM of gear #1
A_1 = pitch radius of gear #1
θ = pressure angle in plane normal to shaft
β = helix angle, where $\tan\beta = \dfrac{\pi \text{ x pitch diameter of gear}}{\text{lead of helix}}$

$$F = \frac{(63{,}025)\text{ H.P.}}{N_1R_1} \text{ lbs.} \qquad (12A)$$

$$F_R = F\tan\theta \text{ lbs.} \qquad (13)$$

$$F_T = F\tan\beta \text{ lbs.} \qquad (14)$$

Note that: (i) The helices on mating gears are of opposite hand:
(ii) The direction of the thrust load is determined by the Condition (see Figure 12) that the vector sum of the radial force and the thrust load is normal to the helix. This implies that reversal of rotation causes reversal of thrust.

The total radial shaft load = $(F^2 + F_R^2)^{1/2}$ for both shafts (15)

The thrust load in the case of helical gears implies that the bearings be capable of carrying both the radial load and the thrust load.

The calculation of the radial bearing load in the case of shafts with two bearings can be obtained from Cases (a) and (b).

Again we note that since action and reaction are equal and opposite, the three orthogonal force components F, F_R and F_T act on both gears (and shafts), but in opposite directions.

(h) Straight Bevel Gears

F = tangential force, perpendicular to plane of shaft axes (lbs).

F_{TG} = thrust force on gear (lbs)
F_{TP} = thrust force on pinion (lbs)
N_P = pinion RPM
R_P = pitch radius of pinion (in).
Ø = normal pressure angle
γ = semi pinion pitch-cone angie
T_P = Number of teeth on pinion
T_G = Number of teeth on gear
H.P. = horsepower

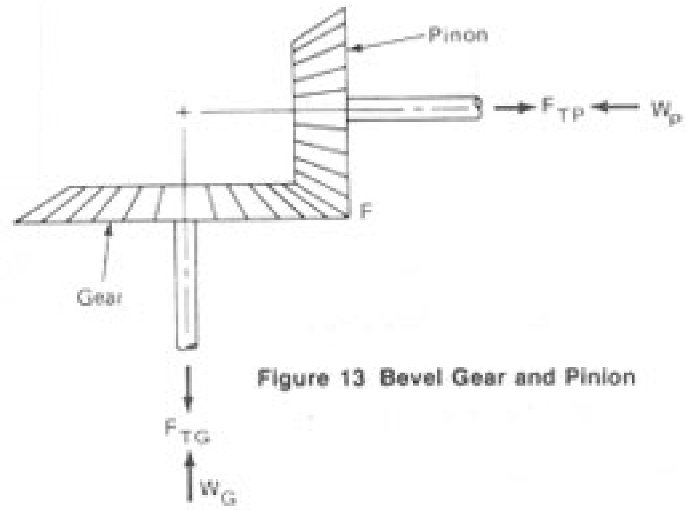


Figure 13 Bevel Gear and Pinion

$$F = \frac{(63{,}025)\ H.P.}{N_P R_P} \qquad (16)$$

$$F_{TG} = F \tan Ø \cos\gamma \qquad (17)$$

$$F_T = F \tan Ø \sin\gamma \qquad (18)$$

$$\gamma = \tan^{-1} (T_P/T_G) \qquad (19)$$

Note that the direction of F (in all gear drives) depends on the direction of rotation of the driving gear. The thrust loads F_{TG} and F_{TP} are components of the tooth separating force, which must be taken up by both pinion and gear bearings. The directions acting on the gear and the pinion are opposite. Total bearing force on each gear is the vector sum of three forces: tangential, gear thrust and pinion thrust. These forces are shown in Figure 14.

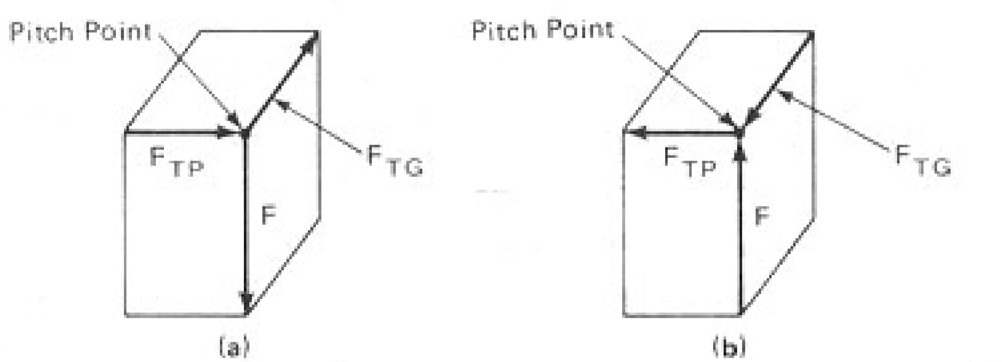


Figure 14 Directions of Force Components on Pinion (a) and Gear (b), assuming pinion is driver with angular-velocity vector, Wp, as shown on Figure 13.

With the aid of these figures the radial bearing toads for shafts with two bearings can be obtained from Cases (a) and (b). The presence of thrust loads again necessitates axial takeup capabilities in the bearings.

(i) Worms and Worm Gears

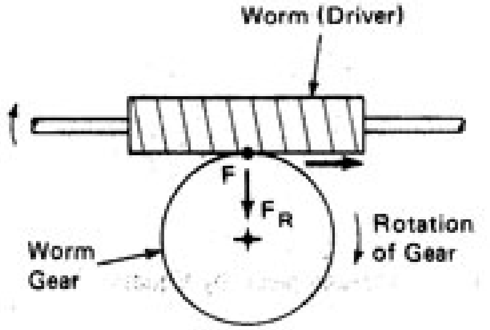


Figure 15 Worm Gear and Worm

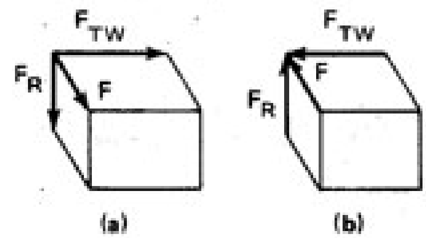


Figure 16 (a) Forces on Gear
(b) Forces on Worm

N_W = RPM of worm
R_W = pitch radius of worm, in
θ = tooth pressure angle
$\gamma = $ lead angle of worm $= \tan^{-1}\left(\frac{L_W}{2\pi R_W}\right)$

$L\omega$ = lead of worm, in

$$F = \frac{(63{,}025)\ H.P.}{N_W R_W} \quad (20)$$

$$F_R = \frac{F \tan\theta}{\tan\gamma} \quad (21)$$

$$F_{TW} = \frac{F}{\tan\gamma} \quad (22)$$

and F = tangential force (tangential to worm, thrust force on gear)
F_R = radial force
F_{TW} = thrust force on worm (acts as tangential force on gear).

Note that the direction of F depends on the direction of rotation of the worm. The three force components, F, F_R and F_{TW} must be taken up by both worm and gear bearings. The directions acting on the worm gear and worm are opposite. Total bearing force on each member is the vector sum of these three forces. With the worm as driver and the gear rotating as shown in Figure 15, the direction of these forces on each member are shown in Figures 16a and b.

With the aid of these figures the radial bearing loads for shafts with two bearings can be obtained from Cases (a) and (b). Once again both thrust and radial forces need to be taken up by the bearings.

(j) Compound Spur-Gear Train

As an example of the bearing-reaction calculations for an entire gear train we consider the spur-gear train shown in Figure 17. The gear train shown in Figure 17 transmits 1/20 horsepower. Shaft S—1 is the driver. if shaft S—2 rotates at 100 rpm CW as shown, what are the bearing reaction forces on Shaft S—2?

The free body diagram of S—2 is shown in Figure 18a, and component forces are shown in Figure 18b. From the horsepower the transmitted forces are obtained as follows:

$$F_{T_1} = \frac{(HP) \cdot 33{,}000}{V} = \frac{(1/20) \cdot 33{,}000}{\pi \cdot 27/24 \cdot 100/12} = 56 \text{ lbs.}$$

$$F_{T_2} = \frac{(1/20) \cdot 33{,}000}{\pi \cdot 10/20 \cdot 100/12} = 126 \text{ lbs.}$$

These transmitted forces are generated from contact tooth forces given by Equation 2:

$$\text{(Contact force)} \quad F_{12} = \frac{F_{T_1}}{\cos \phi} = \frac{56}{\cos 20°} = 59.6 \text{ lbs.}$$

$$\text{(Contact force)} \quad F_{43} = \frac{F_{T_2}}{\cos \phi} = \frac{126}{\cos 20°} = 134.1 \text{ lbs.}$$

Where the double subscripts designate transmission of forces between members. For example, F_{12} means the force of gear 1 on gear 2. The above contact tooth forces plus the bearing reaction forces hold the shaft in equilibrium as pictured in Figure 18a. Resolving all forces into X and V components, as shown in Figure 19, the equilibrium equations can be applied.
Because of the particular shaft orientation given for this problem the X and V components of contact force F_{12} are the tangential and normal components, but this is not true of F_{43} which is inclined 50° to the X axis.

From basic equilibrium equations: $\Sigma F_x = 0$

$$F_{12}^{x} - R_{A}^{x} + F_{43}^{x} - R_{B}^{x} = 0 \qquad (23)$$

$$59.6 \cdot \sin 20° - R_{A}^{x} + 134 \cos 50° - R_{B}^{x} = 20.4 - R_{A}^{x} + 86.1 - R_{B}^{x} = 0$$

$$R_{B}^{x} - R_{A}^{x} = 106.5 \qquad (24)$$

From equilibrium of the Y components: $\Sigma F_y = 0$

$$F_{12}^{y} - R_{A}^{y} + F_{43}^{y} - R_{B}^{y} = 0 \qquad (25)$$

$$59.6 \cdot \cos 20° - R_{A}^{y} + 134 \sin 50° - R_{B}^{y} = 0$$

$$56 - R_{A}^{y} + 102.7 - R_{B}^{y} = 0$$

$$R_{B}^{y} + R_{A}^{y} = 158.7 \qquad (26)$$

Thus, there are 4 unknowns and two equations. However, if the moment equilibrium equations are written, the unknowns can be reduced.

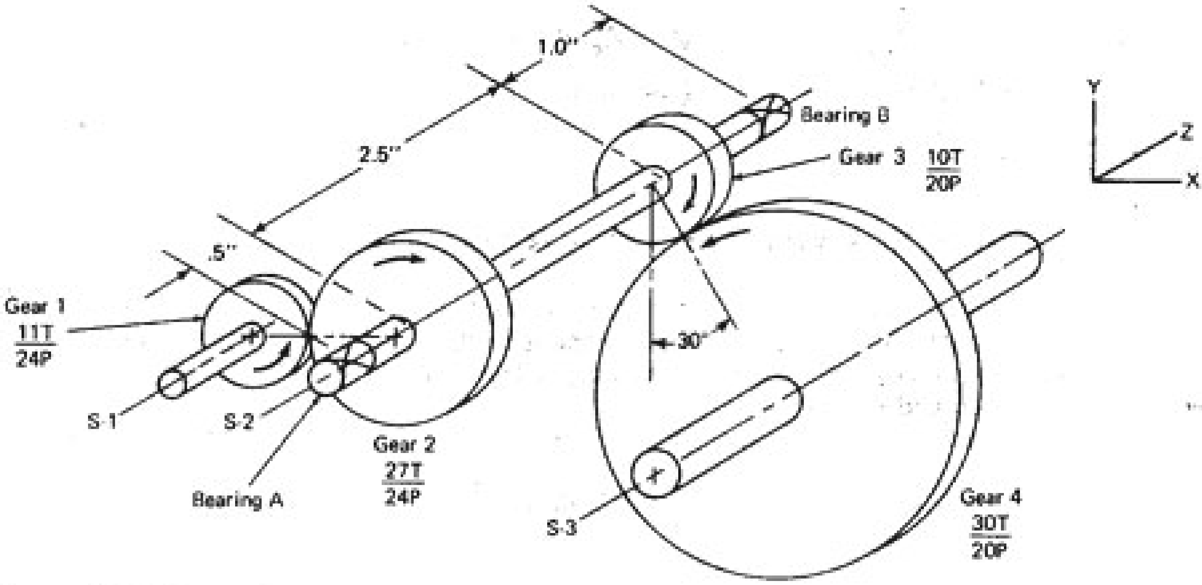
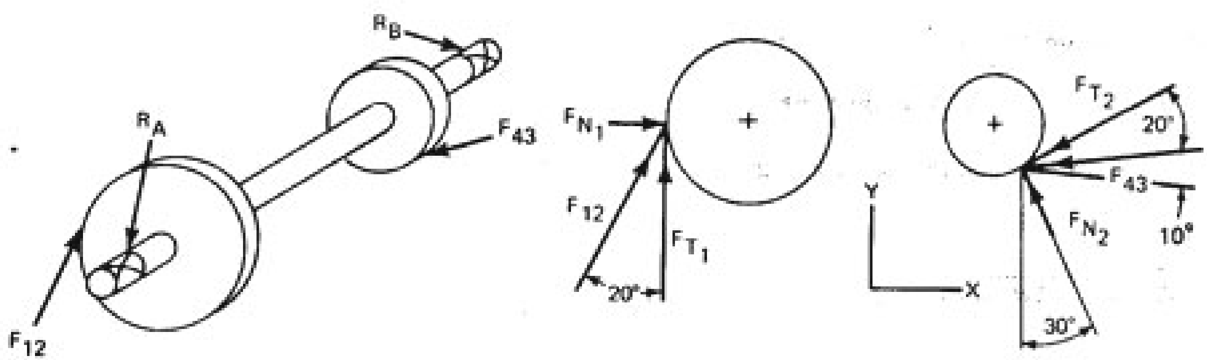


Figure 17 Shaft and Gear Design Details for Calculation Example (Only Gear Pitch Cylinders Shown)



(a) Free Body Diagram of Shaft S–2 (b) Gear Force Orientation and Components of Forces on Shaft S–2

Figure 18 Free Body Diagram and Orientation of Gear Forces

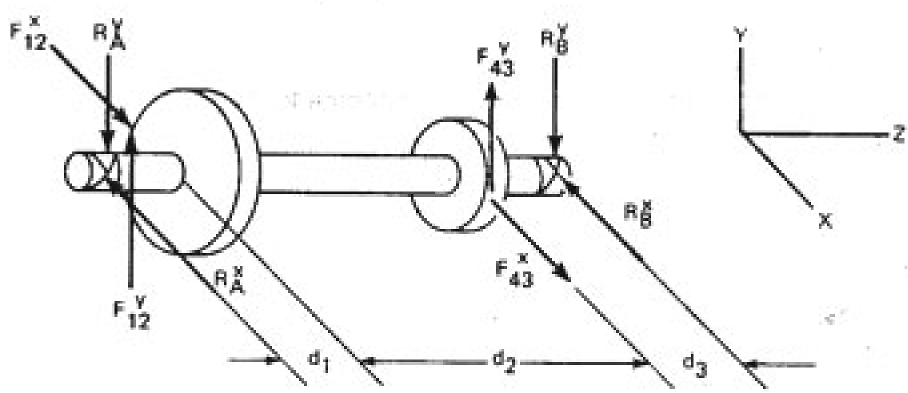


Figure 19 Free Body Diagram of Shafts S–2 with all Forces Shown as X and Y Components

Taking moments about bearing A, first about the X axis and then about the Y axis (using the convention Positive moments are CCW):

$$\Sigma M_A^x = 0 = F_{12}^y \cdot d_1 + F_{43}^y (d_1 + d_2) - R_B^y (d_1 + d_2 + d_3)$$

$$= 56(.5) - 102.7\,(3) - R_B^y\,(4) = 0$$

$$\text{or } R_B^y = \frac{28 + 308.1}{4} = 84.0 \text{ lbs.}$$

and from equation (26) $R_A^y = 158.7 - 84 = R_A^y = 74.7$ lbs.

Note that the sign of R_B^Y is negative. This means its direction is actually opposite of that assumed in equilibrium Equation 26. Thus, in Figure 19 component R_B^Y should be drawn in reversed direction to that shown. Conversely, component R_A^Y has a positive sign, so its' direction assumed for the equilibrium equation and Figure 19 is correct, To determine the X reaction components moments are taken about the Y axis at bearing A:

$$\Sigma M_A^y = 0 = -F_{12}^x d_1 - F_{43}^x (d_1 + d_2) + R_B^x (d_1 + d_2 + d_3) = 0$$

$$= -20.4\,(.5) - 86.1\,(3) + R_B^x\,(4) = 0$$

$$R_B^x = \frac{258.3 + 10.2}{4} = 67.1 \text{ lbs.}$$

and from Equation 24:

$$R_A^x = 106.5 - 67.1 = 39.4 \text{ lbs.}$$

Combining the reaction components:

Bearing A:

$$R_A = \sqrt{(R_A^x)^2 + (R_A^y)^2} = \sqrt{(39.4)^2 + (74.7)^2}$$

$$R_A = 84.5 \text{ lbs.}$$

and the orientation of this force vector comes from:

$$\theta_A = \tan^{-1} \frac{R_A^y}{R_A^x} = \tan^{-1} \frac{74.7}{39.4} = \tan^{-1} 1.896 = 62°12'$$

Since the Y component points down and the X component to the right, the reaction force vector is oriented 109°11' from the positive X axis.

Bearing B:

$$R_B = \sqrt{(R_B^x)^2 + (R_B^y)^2} = \sqrt{(67.1)^2 + (84)^2}$$

$$R_B = 107.5 \text{ lbs.}$$

The orientation of this force vector is:

$$\theta_B = \tan^{-1} \frac{R_B^y}{R_B^x} = \tan^{-1} \frac{84}{67.1} = \tan^{-1} 1.251 = 51°22'$$

The resultant bearing reaction forces and orientations are pictured in Figure 20.