cycles per revolution of the input shaft. This fluctuation, which is accompanied by corresponding angular accelerations, increases with the operating angle and can be as much as 5 of peak angular velocity (in the case of a 30° operating angle). In selecting a joint the effect of these fluctuations on static torque, inertia torque and system performance needs to be kept in mind.

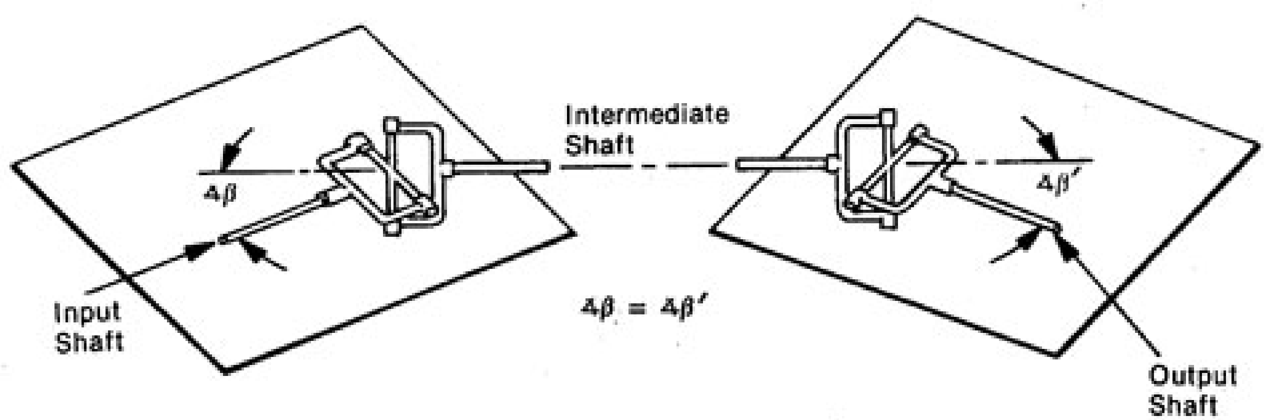

Figure 2 Dual Universal For Elimination of Velocity Variation

The nonuniformity of the transmission can be eliminated by using two appropriately phased universal joints in series, as shown in Figure 2. In such cases the velocity variation induced by one joint can be made to cancel that of the other, thereby transmitting a constant (1:1) angular velocity ratio between shafts. The angular velocity Fluctuation of the intermediate shaft, however, cannot be avoided.

Two universal joints in series also permit coupling of two laterally displaced shafts (single Cardan joints are limited to intersecting shafts).

Single Cardan joints have the following advantages:

- Low side thrust on bearings.
- Large angular displacements are possible.
- High torsional stiffness.
- High torque capacity.

They have the following disadvantages:

- Velocity and acceleration fluctuation increases with operating angle.
- Lubrication is required to reduce wear.
- Shafts must lie in precisely the same plane.
- Backlash difficult to control.

4.0 KINEMATICS

For a uniformly rotating input shaft, the output shaft angular-velocity and angular acceleration undergo two cycles per revolution of the input shaft. The angular displacement of the out' put shaft does not precisely follow that of the input shaft but leads or lags, again with two cycles per revolution.

The angular-velocity variation as a function of operating angle is illustrated in Figure 3.

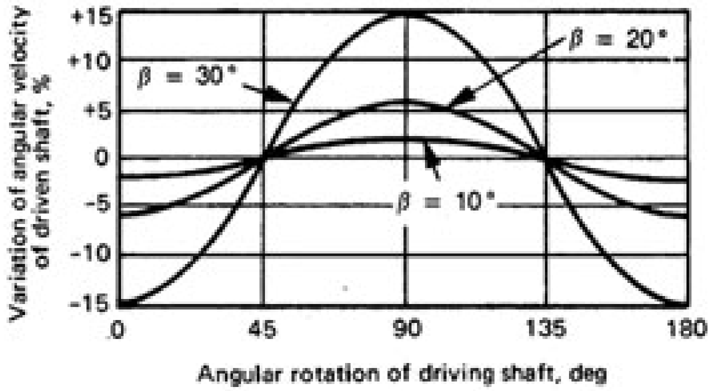

Figure 3 Velocity Ratio Variation With Angle β

The peak displacement lead (or lag), peak angular-velocity ratios (max. and mm.) and peak angular-acceleration ratios are shown as a function of operating angle in Table 1, which is reproduced from “The Analytical Design of Universal Joints” by S.J. Baranyl, Design News, Sept. 1, 1969.

The table should always be consulted for exact numerical values. As a qualitative guideline, It may be kept in mind that for small operating angles (say up to 10), the angular.displacement error (max. lead or lag), the deviation of the max. and mm. angular velocity ratios from unity, and the maximum angular acceleration ratio are very nearly proportional to the square of the operating angle.

The static torque transmitted by the output shaft Is equal to the product of the input torque and the angular.velocity ratio.

The angular acceleration gives rise to an inertia torque, as well as to vibrations. The inertia torque typically would be equal to the sum of the product of the angular acceleration of the output shaft (rad/sec2) and the polar mass moment of inertia of the output shaft (in.lb.sec2) and the output torque (with the units ir.dicated the torque would be given In in.lbs).

The inertia loading often determines the ultimate limit on the speed of operation of the joint. Recommended speed limits vary depending on operating angle, transmitted power and nature .f the application. Recommended peak angular accelerations of the driven shaft vary from 300 radlsec2 to over 2000 rad/sec2 in power drives. In light instrument drives, the allowable accele. rations may be higher. For an accurate determination of allowable speed, a stress determination is necessary.

4.1 Example 1: Determining the Maximum Inertia Torque

A universal joint operates at 250 RPM with an operating angle of 10. Find the max. angular displacement lead (or lag), max. and mm. angular-velocity of output shaft and max. angular acceleration of output shaft.

If the system drives an inertial load so that the total inertial load seen by the output shaft (including its own inertia) can be represented by a steel, circular disc attached to the output shaft (radius r = 3", thickness t = 1/4"), find the max. inertia torque of the drive.

From Table 1 with β = 10, the max. displacement lead (or lag) = 0.439° = 26.3’. The max. and mm. angular-velocity ratios are given as 1.0154 and 0.9848, respectively. Hence, the corresponding output-shaft speeds are:

Q_{max} = (250) (1.0154) = 254 RPM
Q_{min} = (250) (0.9848) = 246 RPM.

The Effect of Shaft Angle (β) on Single Universal Joint Performance For Constant Input Speed*

TABLE 1

Operating Angle Between Shafts (β) Deg.	Maximum Lead or Lag of Output Shaft Displacement (λ), deg. Relative to Input Shaft Displacement	Maximum Angular Velocity Ratio (Ω_{max})	Minimum Angular Velocity Ratio (Ω_{min})	Maximum Angular Acceleration Ratio = $\frac{\alpha_{max}}{\omega^2}$, Where α_{max} = Maximum Angular Acceleration of Output Shaft; ω = Angular Velocity of Input Shaft, rad/sec.
0	0.000	1.0000	1.0000	0.0000
1	0.004	1.0002	0.9998	0.0003
2	0.017	1.0006	0.9994	0.0012
3	0.039	1.0014	0.9986	0.0027
4	0.070	1.0024	0.9976	0.0049
5	0.109	1.0038	0.9962	0.0076
6	0.157	1.0055	0.9945	0.0110
7	0.214	1.0075	0.9925	0.0150
8	0.280	1.0098	0.9903	0.0196
9	0.355	1.0125	0.9877	0.0248
10	0.439	1.0154	0.9848	0.0306
11	0.531	1.0187	0.9816	0.0371
12	0.633	1.0223	0.9781	0.0442
13	0.744	1.0263	0.9744	0.0520
14	0.864	1.0306	0.9703	0.0604
15	0.993	1.0353	0.9659	0.0694
16	1.132	1.0403	0.9613	0.0792
17	1.280	1.0457	0.9563	0.0896
18	1.437	1.0515	0.9511	0.1007
19	1.605	1.0578	0.9455	0.1125
20	1.782	1.0642	0.9397	0.1250
21	1.969	1.0711	0.9336	0.1382
22	2.165	1.0785	0.9272	0.1522
23	2.372	1.0864	0.9205	0.1670
24	2.590	1.0946	0.9135	0.1826
25	2.817	1.1034	0.9063	0.1990
26	3.055	1.1126	0.8988	0.2162
27	3.304	1.1223	0.8910	0.2344
28	3.564	1.1326	0.8829	0.2535
29	3.835	1.1434	0.8746	0.2735
30	4.117	1.1547	0.8660	0.2946
31	4.411	1.1666	0.8572	0.3167
32	4.716	1.1792	0.8480	0.3400
33	5.034	1.1924	0.8387	0.3644
34	5.363	1.2062	0.8290	0.3902
35	5.705	1.2208	0.8192	0.4172
36	6.060	1.2361	0.8090	0.4457
37	6.428	1.2521	0.7986	0.4758
38	6.809	1.2690	0.7880	0.5074
39	7.204	1.2868	0.7771	0.5409
40	7.613	1.3054	0.7660	0.5762