From Table 1. for β 120, we have $\alpha_{max}/w^2 = 0.0442$. The weight, W, of the disc is $\pi r^2 t y$, where $y = 0.283$ lbs/in^3 and denotes the density of steel.

Thus $W = \pi (6)^2 (0.5) (0.283) = 16$ lbs. The polar mass moment of inertia, l, of the disc is given by

$$l = \frac{Wr^2}{2g} = \frac{(16)(16)^2}{(2)(386)} = 0.746 \text{ in-lb-sec}^2.$$

The inertia torque = $l\,\alpha_{max}$ = 50% of 250 in-lbs = 125 in.lbs.

Hence, $0.0442\, w^2 l = 125$, i.e. $(0.746)(0.0442)\, w^2 = 125$,

giving $w = 61.6$ rad/sec. $= \frac{(61.6)(60)}{(2)(\pi)} = 588$ RPM.

Hence, If the inertia torque is not to exceed its limit, the max. speed of the input shaft is 588 RPM.

For joints made with thermoplastic material, consult the catalogue, which Contain design charts for the torque rating of such joints.

6.0 SECONDARY COUPLES

In designing support bearings for the shafts of a Cardan joint and in determining vibrational characteristics of the driven system, it is useful to keep in mind the so-called secondary couples or rocking torques, which occur in universal joints.

These are rocking couples in the planes of the yokes, which tend to bend the two shafts and rock them about their bearings. The bearings are thus cyclIcally loaded at the rate of two cycles per shaft revolution. The maximum values of the rocking torques are as follows:

Max. rocking torque on Input shaft = $T_{in} \tan \beta$;
Max. rocking torque on output shaft = $T_{in} \sin \beta$,

where T_{in} denotes the torque transmitted by the input shaft and β the operating angle. These couples are always 180° out of phase. The bearing force induced by these couples is equal to the magnitude of the rocking couple divided by the distance between shaft bearings.

For example if the Input torque, T_{in}, is 1000 in.lbs. and the operating angle is 20', while the distance between support bearIngs on each shaft is 6' the max. secondary couple acting on the input shaft is (1000) (tan 20°) = 364 in-lbs. and on the output shaft it is (1000) (sin 20°) = 342 in-lbs. The radial bearing load on each bearing of the input shaft is 364/6 = 60.7 lbs. and it is 342/6 = 57 lbs. for each bearing of the output shaft. The bearings should be selected accordingly.

It has been observed also that due to the double frequency of these torques, the critical speeds associated with universal drives may be reduced by up to 50% of the value calculated by the standard formulas for the critical speeds of rotating shafts. The exact percentage Is a complex function of system design and operating conditions.

7.0 JOINTS IN SERIES

As mentioned in paragraph 3, universal joints can be used in series in order to eliminate velocity fluctuations, to connect offset (non-intersecting) shafts, or both. Figure 2 shows a schematic of such an arrangement.

7.1 Phasing

In order to obtain a constant angular-velocity ratio (1:1) between input and Output shafts, proper phasing of the joints is required. This phasing can be described as follows: two cardan joints in series will transmit a constant angular velocity ratio (1:1) between two intersecting or nonintersecting shafts (see Figure 2), provided that the angle between the connected shafts and the intermediate shaft are equal ($\beta = \beta'$) and that when yoke 1 lies in the plane of the Input and intermediate shafts, yoke 2 lies in the plane of the intermediate shaft and the output shaft .

If shafts 1 and 3 intersect, yokes 1 and 2 are coplanar.

When the above phasing has been realized, torsional and inertial excitation is reduced to a minimum. However, inertia excitation will Inevitably remain in the intermediate shaft 2. because this shaft has the angular acceleration of the output shaft of a single universal joint the first of the two joints in series). It is for this reason that guidelines exist limiting the max. angular accleratlons of the intermediate shaft. Depending on the application values between 300 rad/sec² and values In excess of 1000 rad/sec² have been advocated. In light, Industrial drives the allowable speed may be higher. For an accurate determination of allowable speed, a stress determination is necessary

7.2 Example 5: DetermIning the Maximum Speed of an Input Shaft In a Series

In a drive consisting of two universal joints in series, phased so as to produce a constant (1:1) angular velocity ratio between input and output shafts, the angle between the intermediate shaft and input (and output) shaft is 20°. If the max. angular acceleration of the intermediate shaft is not to exceed 1000 rad/sec2. what is the upper limit of the speed of the input shaft?

From Table 1, with $\beta = 20°$, we find $\alpha_{max}/w^2 = 0.1250$.

Since $\alpha_{max} = 1000$ rad/sec²,

$w^2 = (\alpha_{max})(O.125O) = (1000)/(0.1250) = 8000$ rad/sec².

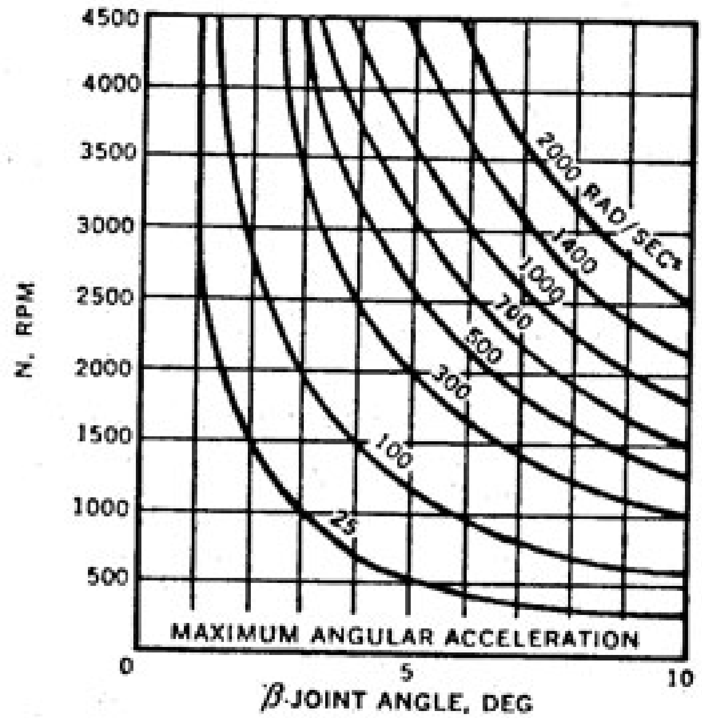
Hence, $w = (8000)^{1/2} = 89.4$ rad/sec. $= \frac{(89A)(60)}{(2)(\pi)} = 854$ RPM.

Hence, the speed of the input shaft should not exceed 854 RPM. When the joint angle is less than or equal to 10°, Table 3 can be used as an alternative.

7.3 Example 6

Same as problem 5, except operating angle is 10°. Here we can use Table 3. The intersection of $\beta = 10°$ and the 1000 sec² curve yields $N \cong 1800$ RPM Hence, the speed of the input shaft should not exceed 1800 RPM. A more exact calculation, as in Example 5. yields N = 1726 RPM. For practical purposes however, the value obtained from Table 3 is entirely satisfactory.

TABLE 3* MAXIMUM ANGULAR ACCELERATION $\left(\frac{rad}{sec^2}\right)$ OF OUTPUT SHAFT OF A SINGLE CARDAN JOINT AS A FUNCTION OF INPUT SPEED (RPM) AND OPERATING ANGLE (DEG.)



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