SECTION 3 DETAILS OF INVOLUTE GEARING

3.1 Pressure Angle

The pressure angle is defined as the angle between the line-of-action (common tangent to the base circles in **Figures 2-3** and **2-4**) and a perpendicular to the line-of-centers. See **Figure 3-1**. From the geometry of these figures, it is obvious that the pressure angle varies (slightly) as the center distance of a gear pair is altered. The base circle is related to the pressure angle and pitch diameter by the equation:

$$d_b = d \cos \alpha \qquad \textbf{(3-1)}$$

where d and α are the standard values, or alternately:

$$d_b = d' \cos \alpha' \qquad \textbf{(3-2)}$$

where d' and α' are the exact operating values.

The basic formula shows that the larger the pressure angle the smaller the base circle. Thus, for standard gears, 14.5° pressure angle gears have base circles much nearer to the roots of teeth than 20° gears. It is for this reason that 14.5° gears encounter greater undercutting problems than 20° gears. This is further elaborated on in **SECTION 4.3**.

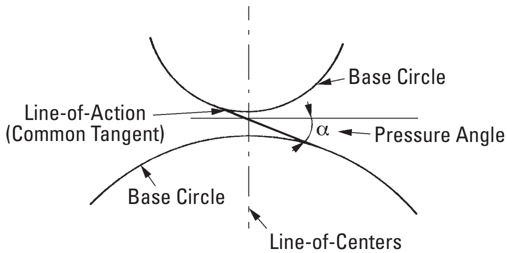

Fig. 3-1 Definition of Pressure Angle

3.2 Proper Meshing And Contact Ratio

Figure 3-2 shows a pair of standard gears meshing together. The contact point of the two involutes, as **Figure 3-2** shows, slides along the common tangent of the two base circles as rotation occurs. The common tangent is called the line-of-contact, or line-of-action.

A pair of gears can only mesh correctly if the pitches and the pressure angles are the same. Pitch comparison can be module (m), circular (p), or base(p_b).

That the pressure angles must be identical becomes obvious from the following equation for base pitch:

$$p_b = \pi\, m \cos \alpha \qquad \textbf{(3-3)}$$

Thus, if the pressure angles are different, the base pitches cannot be identical.

The length of the line-of-action is shown as *ab* in **Figure 3-2**.

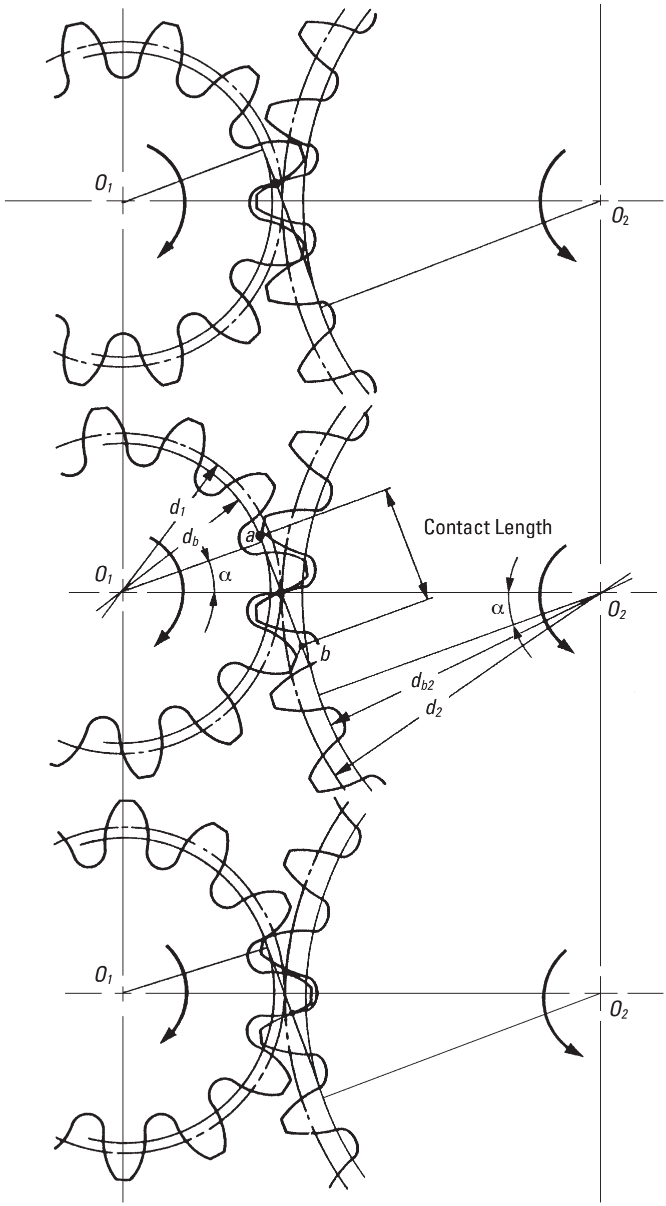

Fig. 3-2 The Meshing of Involute Gear

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3.2.1 Contact Ratio

To assure smooth continuous tooth action, as one pair of teeth ceases contact a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as possible. The measure of this overlapping is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. **Figure 3-3** shows the geometry. The length-of-action is determined from the intersection of the line-of-action and the outside radii. For the simple case of a pair of spur gears, the ratio of the length-of-action to the base pitch is determined from:

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a \sin\alpha}{p \cos\alpha} \qquad \textbf{(3-4)}$$

It is good practice to maintain a contact ratio of 1.2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst-case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high contact ratio generally is not obtained with external spur gears, but can be developed in the meshing of an internal and external spur gear pair or specially designed nonstandard external spur gears.

More detail is presented about contact ratio, including calculation equations for specific gear types, in **SECTION 11**.

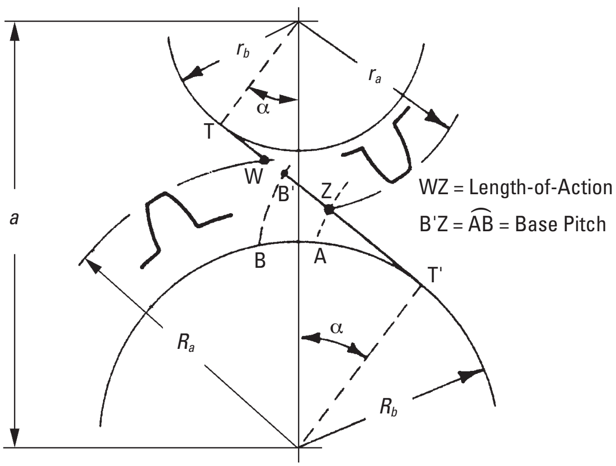

Fig. 3-3 Geometry of Contact Ratio

3.3 The Involute Function

Figure 3-4 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle.

The circle is called the base circle of the involutes. Two opposite hand involute curves meeting at a cusp form a gear tooth curve. We can see, from **Figure 3-4**, the length of base circle arc *ac* equals the length of straight line *bc*.

$$\tan\alpha = \frac{bc}{Oc} = \frac{r_b\theta}{r_b} = \theta \text{ (radian)} \qquad \textbf{(3-5)}$$

The θ in **Figure 3-4** can be expressed as inv $\alpha + \alpha$, then **Formula (3-5)** will become:

$$\text{inv}\,\alpha = \tan\alpha - \alpha \qquad \textbf{(3-6)}$$

Function of α, or inv α, is known as involute function. Involute function is very important in gear design. Involute function values can be obtained from appropriate tables. With the center of the base circle *O* at the origin of a coordinate system, the involute curve can be expressed by values of *x* and *y* as follows:

$$\left.\begin{aligned} x &= r\cos(\text{inv}\,\alpha) = \frac{r_b}{\cos\alpha}\cos(\text{inv}\,\alpha) \\ y &= r\sin(\text{inv}\,\alpha) = \frac{r_b}{\cos\alpha}\sin(\text{inv}\,\alpha) \end{aligned}\right\} \qquad \textbf{(3-7)}$$

where, $r = \dfrac{r_b}{\cos\alpha}$

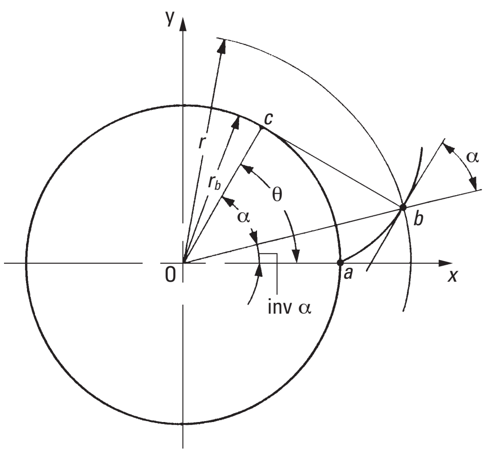

Fig. 3-4 The Involute Curve