SECTION 4 SPUR GEAR CALCULATIONS

4.1 Standard Spur Gear

Figure 4-1 shows the meshing of standard spur gears. The meshing of standard spur gears means pitch circles of two gears contact and roll with each other. The calculation formulas are in **Table 4-1**.

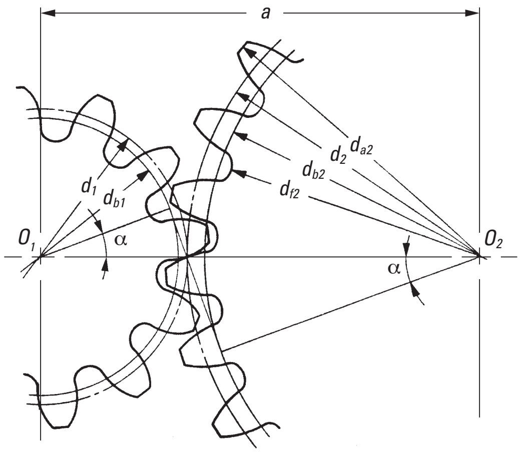

Fig. 4-1 The Meshing of Standard Spur Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = x_2 = 0$)

Table 4-1 The Calculation of Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2 *		12	24
4	Center Distance	a	$\frac{(z_1 + z_2)m}{2}$ *	54.000	
5	Pitch Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	$d \cos \alpha$	33.829	67.658
7	Addendum	h_a	$1.00m$	3.000	
8	Dedendum	h_f	$1.25m$	3.750	
9	Outside Diameter	d_a	$d + 2m$	42.000	78.000
10	Root Diameter	d_f	$d - 2.5m$	28.500	64.500

*The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear.

All calculated values in **Table 4-1** are based upon given module (m) and number of teeth (z_1 and z_2). If instead module (m), center distance (a) and speed ratio (i) are given, then the number of teeth, z_1 and z_2, would be calculated with the formulas as shown in **Table 4-2**.

Table 4-2 The Calculation of Teeth Number

No.	Item	Symbol	Formula		Example	
1	Module	m			3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			0.8	
4	Sum of No. of Teeth	$z_1 + z_2$	$\frac{2a}{m}$		36	
5	Number of Teeth	z_1, z_2	$\frac{i(z_1+z_2)}{i+1}$	$\frac{(z_1+z_2)}{i+1}$	16	20

Note that the numbers of teeth probably will not be integer values by calculation with the formulas in **Table 4-2**. Then it is incumbent upon the designer to choose a set of integer numbers of teeth that are as close as possible to the theoretical values. This will likely result in both slightly changed gear ratio and center distance. Should the center distance be inviolable, it will then be necessary to resort to profile shifting. This will be discussed later in this section.

4.2 The Generating Of A Spur Gear

Involute gears can be readily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also accomplished with gear type cutters using a shaper or planer machine.

Figure 4-2 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear.

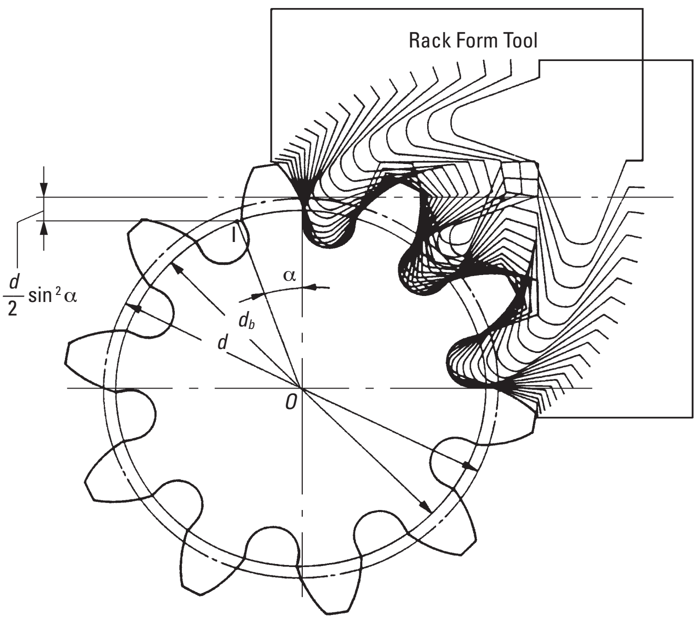

Fig. 4-2 The Generating of a Standard Spur Gear
($\alpha = 20°, z = 10, x = 0$)

4.3 Undercutting

From **Figure 4-3**, it can be seen that the maximum length of the line-of-contact is limited to the length of the common tangent. Any tooth addendum that extends beyond the tangent points (T and T') is not only useless, but interferes with the root fillet area of the mating tooth. This results in the typical undercut tooth, shown in **Figure 4-4**. The undercut not only weakens the tooth with a wasp-like waist, but also removes some of the useful involute adjacent to the base circle.

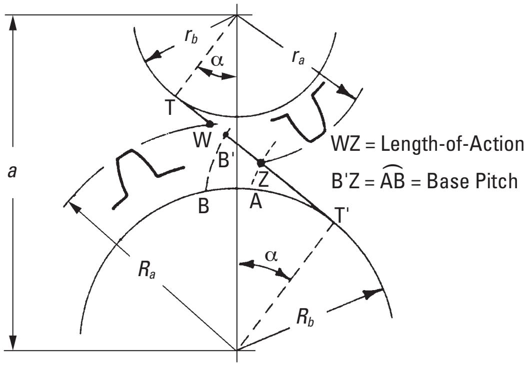

Fig. 4-3 Geometry of Contact Ratio

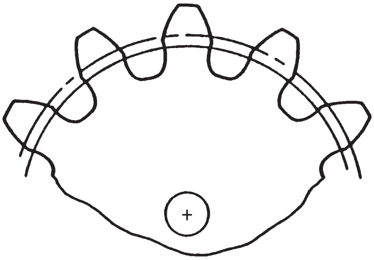

Fig. 4-4 Example of Undercut Standard Design Gear
(12 Teeth, 20° Pressure Angle)

From the geometry of the limiting length-of-contact (T-T', **Figure 4-3**), it is evident that interference is first encountered by the addenda of the gear teeth digging into the mating-pinion tooth flanks. Since addenda are standardized by a fixed value ($h_a = m$), the interference condition becomes more severe as the number of teeth on the mating gear increases. The limit is reached when the gear becomes a rack. This is a realistic case since the hob is a rack-type cutter. The result is that standard gears with teeth numbers below a critical value are automatically undercut in the generating process. The condition for no undercutting in a standard spur gear is given by the expression:

$$\left.\begin{array}{l} \text{Max addendum} = h_a \le \dfrac{mz}{2}\sin^2\alpha \\ \text{and the minimum number of teeth is:} \\ z_c \ge \dfrac{2}{\sin^2\alpha} \end{array}\right\} \quad \textbf{(4-1)}$$

This indicates that the minimum number of teeth free of undercutting decreases with increasing pressure angle. For 14.5° the value of z_c is 32, and for 20° it is 18. Thus, 20° pressure angle gears with low numbers of teeth have the advantage of much less undercutting and, therefore, are both stronger and smoother acting.

4.4 Enlarged Pinions

Undercutting of pinion teeth is undesirable because of losses of strength, contact ratio and smoothness of action. The severity of these faults depends upon how far below z_c the teeth number is. Undercutting for the first few numbers is small and in many applications its adverse effects can be neglected.

For very small numbers of teeth, such as ten and smaller, and for high-precision applications, undercutting should be avoided. This is achieved by pinion enlargement (or correction as often termed), wherein the pinion teeth, still generated with a standard cutter, are shifted radially outward to form a full involute tooth free of undercut. The tooth is enlarged both radially and circumferentially. Comparison of a tooth form before and after enlargement is shown in **Figure 4-5**.

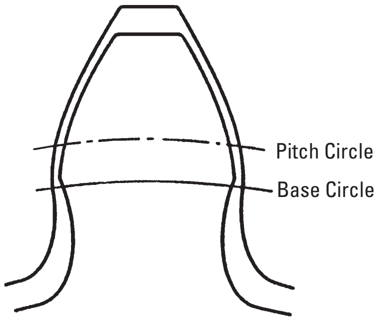

Fig. 4-5 Comparison of Enlarged and Undercut Standard Pinion
(13 Teeth, 20° Pressure Angle, Fine Pitch Standard)

4.5 Profile Shifting

As **Figure 4-2** shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in **Figure 4-6**, can prevent undercut.

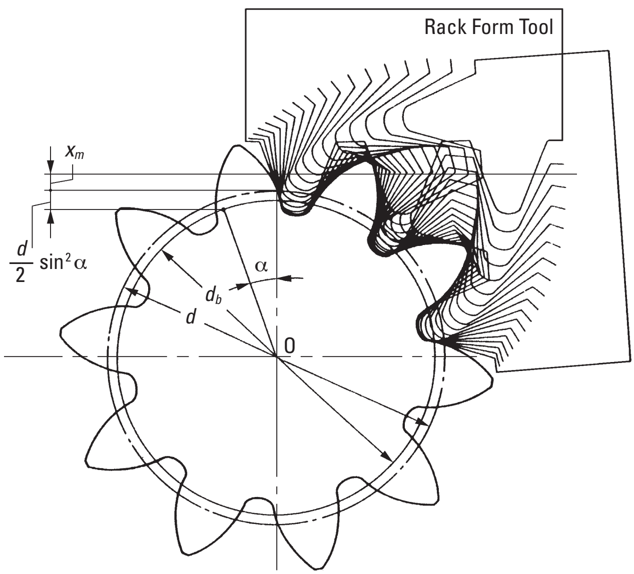

Fig. 4-6 Generating of Positive Shifted Spur Gear
(α = 20°, z = 10, x = +0.5)

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Undercutting will get worse if a negative correction is applied. See **Figure 4-7**.

The extra feed of gear cutter (xm) in **Figures 4-6** and **4-7** is the amount of shift or correction. And x is the shift coefficient.

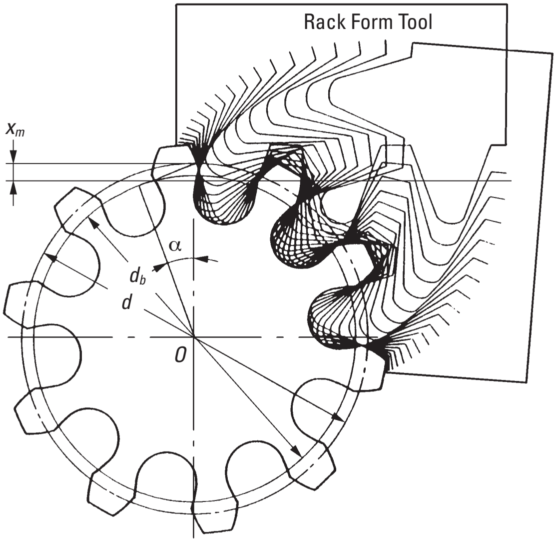

Fig. 4-7 The Generating of Negative Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = -0.5$)

The condition to prevent undercut in a spur gear is:

$$m - xm \leq \frac{zm}{2} \sin^2 \alpha \quad \textbf{(4-2)}$$

The number of teeth without undercut will be:

$$z_c = \frac{2(1 - x)}{\sin^2 \alpha} \quad \textbf{(4-3)}$$

The coefficient without undercut is:

$$x = 1 - \frac{z_c}{2} \sin^2 \alpha \quad \textbf{(4-4)}$$

Profile shift is not merely used to prevent undercut. It can be used to adjust center distance between two gears.

If a positive correction is applied, such as to prevent undercut in a pinion, the tooth thickness at top is thinner.

Table 4-3 presents the calculation of top land thickness.

Table 4-3 The Calculations of Top Land Thickness

No.	Item	Symbol	Formula	Example
1	Pressure angle at outside circle of gear	α_a	$\cos^{-1}\left(\frac{d_b}{d_a}\right)$	$m = 2$, $\alpha = 20°$, $z = 16$, $x = +0.3$, $d = 32$, $d_b = 30.07016$, $d_a = 37.2$, $\alpha_a = 36.06616°$, $\text{inv}\,\alpha_a = 0.098835$, $\text{inv}\,\alpha = 0.014904$, $\theta = 1.59815°$ (0.027893 radian), $s_a = 1.03762$
2	Half of top land angle of outside circle	θ	$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\text{inv}\,\alpha - \text{inv}\,\alpha_a)$ (radian)	
3	Top land thickness	s_a	θd_a	

4.6 Profile Shifted Spur Gear

Figure 4-8 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d_w)and the working (operating) pressure angle (α_w). These values are obtainable from the operating (or i.e., actual) center distance and the following formulas:

$$\left.\begin{aligned} d_{w1} &= 2a_x \frac{z_1}{z_1 + z_2} \\ d_{w2} &= 2a_x \frac{z_2}{z_1 + z_2} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a_x}\right) \end{aligned}\right\} \qquad (4\text{-}5)$$

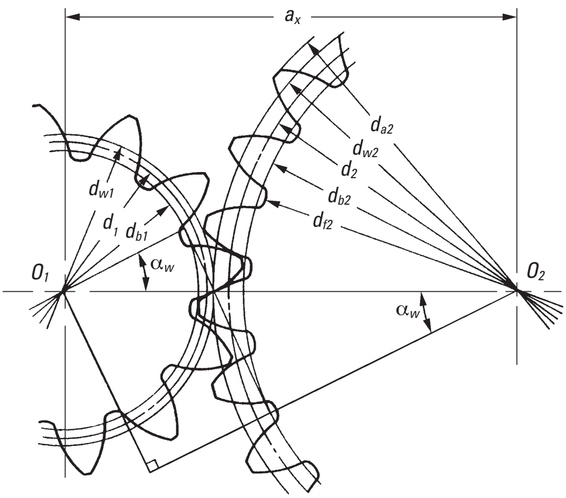

Fig. 4-8 The Meshing of Profile Shifted Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

In the meshing of profile shifted gears, it is the operating pitch circles that are in contact and roll on each other that portrays gear action. The standard pitch circles no longer are of significance; and the operating pressure angle is what matters.

A standard spur gear is, according to **Table 4-4**, a profile shifted gear with 0 coefficient of shift; that is, $x_1 = x_2 = 0$.

Table 4-4 The Calculation of Positive Shifted Gear (1)

No.	Item	Symbol	Formula	Example: Pinion	Example: Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		12	24
4	Coefficient of Profile Shift	x_1, x_2		0.6	0.36
5	Involute Function α_w	inv α_w	$2 \tan \alpha \left(\frac{x_1 + x_2}{z_1 + z_2}\right) + \text{inv}\,\alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886°	
7	Center Distance Increment Factor	y	$\frac{z_1 + z_2}{2}\left(\frac{\cos \alpha}{\cos \alpha_w} - 1\right)$	0.83329	
8	Center Distance	a_x	$\left(\frac{z_1 + z_2}{2} + y\right) m$	56.4999	
9	Pitch Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d \cos \alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos \alpha_w}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1 + y - x_2)m$ $(1 + y - x_1)m$	4.420	3.700
13	Whole Depth	h	$[2.25 + y - (x_1 + x_2)]m$	6.370	
14	Outside Diameter	d_a	$d + 2h_a$	44.840	79.400
15	Root Diameter	d_f	$d_a - 2h$	32.100	66.660

Table 4-5 is the inverse formula of items from 4 to 8 of **Table 4-4**.

Table 4-5 The Calculation of Positive Shifted Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		56.4999	
2	Center Distance Increment Factor	y	$\frac{a_x}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_1 + z_2)\cos \alpha}{2y + z_1 + z_2}\right]$	26.0886°	
4	Sum of Coefficient of Profile Shift	$x_1 + x_2$	$\frac{(z_1 + z_2)(\text{inv}\,\alpha_w - \text{inv}\,\alpha)}{2 \tan \alpha}$	0.9600	
5	Coefficient of Profile Shift	x_1, x_2		0.6000	0.3600

There are several theories concerning how to distribute the sum of coefficient of profile shift, $(x_1 + x_2)$ into pinion, (x_1) and gear, (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

4.7 Rack And Spur Gear

Table 4-6 presents the method for calculating the mesh of a rack and spur gear. **Figure 4-9a** shows the pitch circle of a standard gear and the pitch line of the rack.

One rotation of the spur gear will displace the rack (l) one circumferential length of the gear's pitch circle, per the formula:

$$l = \pi m z \qquad \textbf{(4-6)}$$

Figure 4-9b shows a profile shifted spur gear, with positive correction *xm*, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount *xm*. Also, the pitch line of the rack has shifted outward by the amount *xm*.

Table 4-6 presents the calculation of a meshed profile shifted spur gear and rack. If the correction factor x_1 is 0, then it is the case of a standard gear meshed with the rack.

The rack displacement, l, is not changed in any way by the profile shifting. **Equation (4-6)** remains applicable for any amount of profile shift.

Table 4-6 The Calculation of Dimensions of a Profile Shifted Spur Gear and a Rack

No.	Item	Symbol	Formula	Example: Spur Gear	Example: Rack
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z		12	—
4	Coefficient of Profile Shift	x		0.6	—
5	Height of Pitch Line	H		—	32.000
6	Working Pressure Angle	α_w		20°	
7	Center Distance	a_x	$\frac{zm}{2} + H + xm$	51.800	
8	Pitch Diameter	d	zm	36.000	—
9	Base Diameter	d_b	$d \cos \alpha$	33.829	—
10	Working Pitch Diameter	d_w	$\frac{d_b}{\cos \alpha_w}$	36.000	—
11	Addendum	h_a	$m(1 + x)$	4.800	3.000
12	Whole Depth	h	$2.25m$	6.750	
13	Outside Diameter	d_a	$d + 2h_a$	45.600	—
14	Root Diameter	d_f	$d_a - 2h$	32.100	—